along the kernel radius. The secondary shock wave is initially so weak that it is swept outward by the expanding flow but it gains strength and, as shown by Friedman,[4] at about the time that the rarefaction fan reaches the origin the secondary shock wave is moving inward, even in stationary coordinates. This shock wave is initially quite weak so that it reaches the origin about 0.1 μsec after the rarefaction fan is reflected. The reflected shock front now establishes an order of magnitude variation of temperature along the radius, but this time the maximum is at the origin. Successive reflections and interactions with the contact surface reverse the temperature gradients along the kernel radius with a period of about twice the characteristic time of the problem. The assumption of uniform kernel temperature cannot be justified. The power law and exponential expression for the temperature dependence of the reaction rates magnify the order of magnitude errors introduced by the incorrect gas dynamics model assumed by Ref. 1 and thus, the whole calculation is meaningless.

A further point which, although relatively minor compared to the fundamental objections raised above, nevertheless deserves mention in order to view the work of Zajac and Oppenheim in the proper perspective. In the calculations of the flowfield ahead of the expanding spherical kernels, Ref. 1 did not predict the existence of the primary shock wave at the head of the disturbed flow. The absence of the shock wave ahead of the accelerating contact surface was dismissed with the statement that "Shock waves were generated only in the case of plane and cylindrical flows, the increase of the cross-sectional area in the spherical case having been evidently too large for this purpose." The inability to calculate a shock wave suggests numerical difficulties; moreover, the accompanying statement cannot be accepted since it contradicts directly the discussion on p. 428 of Courant and Friedrichs, who demonstrate clearly that even a uniformly expanding sphere generates a shock wave.

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Reply by Authors to A. Wortman

A. K. OPPENHEIM*
University of California, Berkeley, Calif.

AND

L. J. ZAJAC†
North American Rockwell Corporation, Canoga Park, Calif.

EVIDENTLY, the commentator fell a victim to many misconceptions. Nonetheless the authors appreciate his comments for they bring up a number of interesting points that might have been indeed missed by a casual reader.

The basic feature of our studies has been described by him quite correctly at the end of the first paragraph although at the start he accused us of "failing to recognize the principal features of transient ... flows." He then went on to describe the well-known solutions of the so-called spherical or cylindrical shock tube problem where, as it was first shown by Wecken in 1950 (Ref. 1), a secondary imploding shock wave is formed. As succinctly phrased by Friedman (Wortman's Ref. 4), "the physical reason for the secondary shock formation is that the high-pressure gas, upon passing through a spherical rarefaction wave, must expand to lower pressures than those reached through an equivalent one-dimensional expansion." Finally he pointed out, with reference to the text of Courant and Friedrichs, that even a uniformly expanding sphere generates a shock wave—a fact that has been well established since the first classical paper in blast wave theory written by G. I. Taylor in 1939 and published in 1946 (Ref. 2).

It is the lack of the evidence in our results of the existence of the secondary shock and, in the spherical case, of even the primary shock that forms the major issue in the critique of the commentator. What he evidently overlooked was the fact that, in all the cases to which he referred, the time for the formation of the shock was negligibly small in comparison to the scale of the phenomenon under study. The existence of the shock has to be, in fact, postulated in these cases, in order to satisfy the dynamic compatibility conditions for the whole flowfield.

In contrast to this, our study is concerned with the phenomena occurring within the same order of magnitude in time as that of shock wave formation, belonging therefore to a different class of problems, namely that which, to quote another classical reference, has been studied in the 1940's by Friedrichs[3] who concluded significantly: "Thus it is clear that, in detail, the formation of the shock, at least in its initial stages, depends in a very sensitive way on the motion of the piston which produces the wave" (p. 238 of Ref. 3).

Our studies were then of a pioneering character in one major respect, namely that, as Wortman recognized, they provided information on the motion of the "piston" associated with a given kinetic scheme of exothermic chemical reactions. In order to be able to take properly into account the chemical kinetic and the thermodynamic properties of the reacting system, we had to

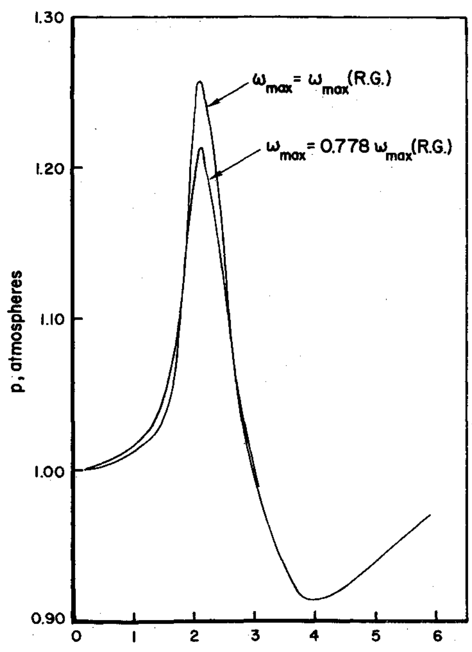

Fig. 1 Pressure pulse computed for a bulk expansion model in perfect gases with constant specific heats for a given exothermic power pulse.

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* Professor. Fellow AIAA.

† Research Engineer. Associate Fellow AIAA.

introduce, vast simplifications in the gas-dynamic description of the system. What we got out of this was the law on the rate at which the heat is released, or the time profile of the exothermic power pulse, under such idealized conditions.

With this information thus available, we could check the validity of our assumptions. This has been done subsequently to the publication of our paper, yielding results which the commentator may consider as obtained by "exact calculations" whose lack he deplored in his comments. Shortly, what we have done was to formulate the law for an equivalent exothermic power pulse that would produce in a perfect gas with constant specific heats the same pressure pulse and the same "piston" motion of the interface as the reacting gas, still under the idealization of a bulk expansion model. The result of such a fit is shown in Fig. 1‡ with respect to the particular initial conditions in a stoichiometric hydrogen-oxygen mixture as specified in the footnote. The proper pressure pulse is that corresponding to 77·8% of the power peak, ω_{max}, computed for the reacting gas (R.G.). Knowing then the function $\omega = \omega(t)$ describing the time profile of the power pulse, we solved the problem by the use of, essentially, the same method of characteristics that led Wecken[1] to the discovery of the imploding shock. With the perfect gas behavior of the substance now admissible, the problem is simply that of the solution of a two-dimensional, so-called "diabatic," flowfield for which the law of energy release is given. The results are presented in Fig. 2.§ There is a significant pressure profile in space established in the kernel, the pressure at the centerline being significantly higher than that at the interface. Although this is in basic contradiction to the basic assumption of the bulk expansion model, the pressure pulse at the interface is quite similar in the two cases, as it can be seen by comparing the corresponding pressure profiles in Figs. 1 and 2, while, at the same time the trajectories of the interface in the time-space domain were found to be identical (as e.g., Fig. 4 of our paper).

Since the dynamic action of the reacting kernel on the surroundings is fully specified in terms of the pressure pulse at the interface and its motion, one can thus conclude that the bulk expansion model offers, for the purpose of evaluating the flowfield in the surroundings, a good approximation. In particular there was no physical reason, existing in the flowfield as that quoted here earlier from the paper of Friedman, nor computed

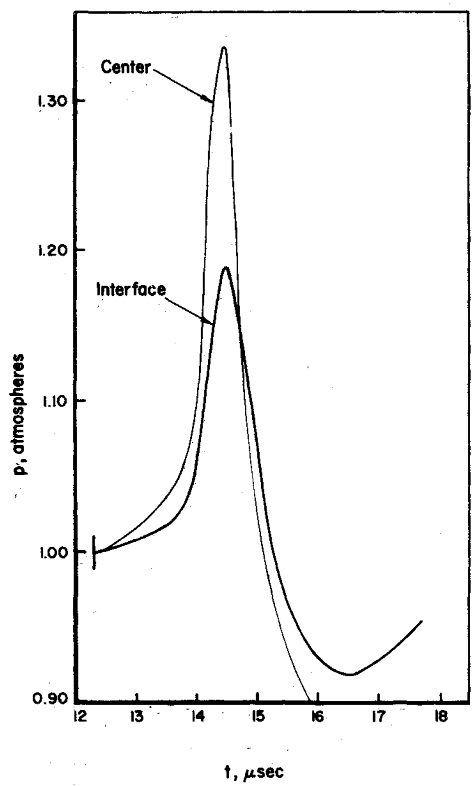

Fig. 2 Pressure pulses at the center line and at the interface computed by the method of characteristics.

by the method of characteristics, for the formation of a secondary shock.

Finally it should be pointed out that the reason why a shock may not be formed at all in our system is that the piston first accelerates and then decelerates, without ever attaining the steady state of constant velocity of propagation—a necessary condition for the existence of the shock front under circumstances appropriate for the Taylor self-similarity solution the commentator must have had in mind in his closing statement.

‡ In Fig. 1, the pressure pulse for $\omega_{max} = 0.778\omega_{max}$ (R.G.) evaluated for perfect gases whose specific heat ratios are 1.35 and 1.14 for the surroundings and the kernel, respectively, matches that obtained from the "real gas" (R.G.) computations for a spherically symmetric flowfield, taking fully into account the chemical kinetic and thermodynamic properties of a stoichiometric hydrogen-oxygen mixture at an initial pressure of 1 atm and an initial temperature of 1200°K. The initial radius of the kernel was 1 mm.

§ In Fig. 2, the specific heat ratios are 1.35 and 1.14 for the surroundings and the kernel, respectively: the exothermic power pulse is $\omega = 0.778\omega$ (R.G.).

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